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Analytical models for cycle time and throughput evaluation of multi-shuttle deep-lane AVS/RS

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Abstract The increased need for just-in-time delivery of finite goods has pulled the development of novel automation solutions to manage warehouse activities. Among the available technologies, Autonomous Vehicle Storage and Retrieval Systems (AVS/RS) rely on light vehicles able to travel independently and to perform different tasks at the same time, thus exhibiting enhanced flexibility and increased throughput level. Nonetheless, techniques to evaluate the performance of such systems still exhibit some gaps and are mainly focused on simple configurations. This paper aims to extend the state of the art by introducing novel analytical models capable to assess the performance of a tier-to-tier, multi-shuttle AVS/RS feeding a deep-lane rack. The proposed approach enables to evaluate the expected cycle time and throughput by (i) enabling the possibility to consider the real criteria adopted to store and retrieve items, and (ii) taking into account the ability of the vehicles to simultaneously perform different tasks. The model is validated against simulations performed on different rack layouts, on AVS/RS with different fleet composition, for different types of cycle. The developed model aims to support both the design and the deployment phases of AVS/RS by enabling quick and accurate performance estimation in a wide variety of scenarios.

Keywords Automated Warehouses · Autonomous vehicle storage and retrieval system · Deep-Lane System · Cycle time · Analytical Modelling · Performance Analysis

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1 Introduction

In recent times, a renewed consideration for warehouses efficiency has arisen. The motivation for this interest is twofold. On the one side, warehouses have to support just-in-time production: both raw materials and intermediate components must be available at the right position in the process at the right time. On the other side, customers require complex product varieties with short lead times and with highly variable demand. To deal with these challenges, great efforts have been spent to develop innovative technologies for warehouse management. Automated systems play a key role in this field: a research made by Markets And Markets [1] stated that the overall market for such solutions will overcome USD 9 Billion by 2023, with an estimated CAGR greater than 7% between 2017 and 2023. This trend is also contributed by the success of businesses mainly consisting in appropriate warehousing activities, such as e-commerce.

The most common automation system for warehouses is represented by Automated Storage and Retrieval Systems (AS/RS), which mainly consist of a set of stacker cranes able to perform movements along a given aisle as well as to store and retrieve a unit load (UL) in the rack. Despite their popularity and the relatively low implementation cost, such systems exhibit low flexibility: ULs are processed one-by-one, with limited capability to manage a variable pace. Further, AS/RS can feed only single- or double-depth racks: therefore, the number of aisles increases with the size of the warehouse, leading to under-exploiting the available space.

An alternative system with enhanced performances has been developed in the last decade. The underlying idea was the replacement of the crane with a lift and the adoption of lighter and compact shuttles to perform material handling. Such technology has been named Autonomous Vehicle Storage and Retrieval System (AVS/RS) [2]. Racks supporting such systems can be made by an arbitrary number of tiers: each of them has a single cross aisle to provide access to the channels which, in turn, can have arbitrary depth. The latter capability enables to collect ULs with common features (e.g. product type, lot number, expiration date, customer) in the same lane to ease the retrieval operations. The reduced number of aisles, compared to traditional systems, enables to better exploit the available space and/or to use smaller, cheaper buildings. AVS/RS have been successfully implemented in different fields, such as food and beverage (even in cold applications, where the improved space efficiency enables to reduce the cost of cooling), tobacco, pharmaceutics, semi-finite materials (e.g. rolls of textiles or papers). Further, they have been found to perform better than the AS/RS from an environmental perspective, due to the greater energy efficiency per cycle [3]. A schematic representation of an AVS/RS and the supported rack is presented in Fig. 1.

Nevertheless, the initial investment for designing and setting up an AVS/RS is higher than a traditional system. Therefore, accurate models for performance estimation are necessary to ensure that the investment may provide the user with the expected performance level. A first step in this direction has been made by the recently issued standard FEM 9.860 [4]; however, the cycles defined in this document do not fully exploit all the technical capabilities of autonomous vehicles yet. Therefore, this work aims to extend the state-of-the art by defining novel models for AVS/RS performance evaluation able to consider as much as possible the technological strengths of such systems. In particular, the models presented in this paper enable to assess the performance of an AVS/RS made of an arbitrary number of shuttles and feeding a rack with arbitrary depth. The natural users of such models are the designers and the adopters of AVS/RS, who need to estimate system performance in realistic operating scenarios.

The remainder of the paper is organized as follows. A review of the existing techniques for AVS/RS performance evaluation is discussed in Section 2. The operating principles of such system are described in Section 3. Then, the original work of this paper is described: the variables and the analytical model are presented in Sections 4 and 5, respectively. The validation methodology and the results are discussed in Sections 6 and 7. Finally, conclusive remarks and perspective works are presented in Section 8.

2 Background

In scientific literature, the first research works concerning AVS/RS have been issued in the early 2000s. A preliminary classification can be done to distinguish tier-to-tier and tier-captive systems. In the former configuration, vehicles can move through different tiers through a lifting table; conversely, in the second case the vehicles are assigned to a given tier that is not changed during the AVS/RS operations.

The first scientific work on AVS/RS is dated 2002: Malmborg [2] presented an analytical model to assess tier-to-tier configurations based on rack topology and vehicles features. The targeted performance indicators were vehicles utilization, cycle time and system throughput. In this work, both Single Command (SC) and Dual Command (DC) cycles were considered. This classification is based on the number of ULs involved in the cycle, which is equal to 1 for SC cycles (i.e. one storage or one retrieval) and 2 for DC cycles (corresponding to

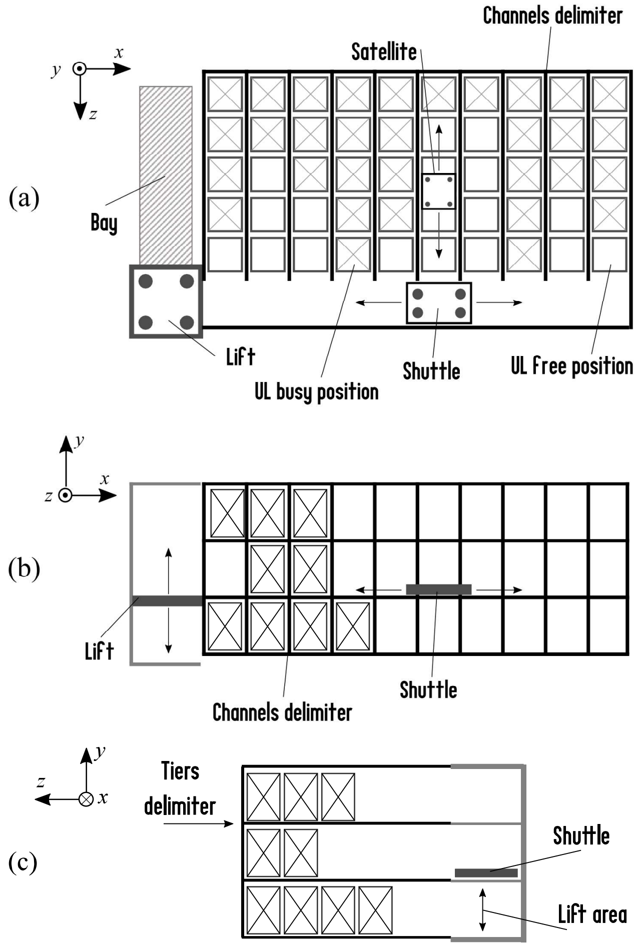

Fig. 1 Representation of the warehouse top (a), front (b), and side (c) views and the AVS/RS studied in the present paper.

both one storage and one retrieval). Later, Malmborg [5] also developed a mathematical model to evaluate the optimal proportion of DC cycles to be performed in order to match with the demand of storage and retrieval tasks.

Another investigated approach relies on queueing theory. Kuo et al. [6] focused on SC cycles and developed a model capable to estimate cycle time and vehicles utilization. Fukunari and Malmborg [7] enriched the model by taking into account the capability of opportunistic pairing of storage and retrieval to improve the overall system performance. They also developed a network queueing approach to reduce as much as possible the computational cost of the model [8]. Zhang et al. [9] used approximation techniques to keep analytical simplicity for a model with non-Poissonian arrival rates and service times. They evaluated both average values and variances for the cycle time. Roy et al. [10] adopted the semi-open queueing network approach: they modeled the lift and the vehicles as mutually interacting independent queues. This methodology initially led to model single-tier systems; then, it has been extended to describe multi-tier racks [11,12]. Epp et al. [13] adopted the open queue network model to assess the performance of a tier-captive, single aisle AVS/RS. Ekren and Heragu [14] developed a regression, simulation-based model to estimate the average cycle time as a function of rack topology and vehicles performance. Ekren [15] deployed simulation models to support designers' decisions in comparing different AVS/RS configuration and the corresponding cost. A hybrid approach made of analytical techniques and queues network has been presented by Marchet et al. [16] to estimate the average cycle time as the sum of travelling and waiting times.

A technology similar to AVS/RS is named Shuttle-Based Storage and Retrieval System (SBS/RS). Such systems are mainly devoted to mini-load warehouses, but exhibit similar difficulties in performance evaluation. They have been first investigated by Carlo and Vis [17]. Analytical models have been introduced by Lerher et al. [18] to evaluate the average duration of SC and DC cycles in single-depth racks. This approach has been extended to take into account also double-depth racks [19] and to model systems where a tier-captive storage/retrieval machine can change aisle [20]. Ekren et al. [21] developed a mathematical model to estimate SBS/RS travel time average and variance, and the mean amount of energy consumption. A method to optimize throughput time, total cost, and energy consumption is presented in [22]. Simulation methods have been used by Ning et al. [23] to evaluate the performance of a tier-captive SBS/RS with multiple elevators in SC cycles. The developed tool is able to self-generate and evaluate different alternative system compositions. Lerher et al. [24] also developed a simulation tool for SBS/RS performance evaluation and studied the relationship between travel times and system throughput [25]. Ekren [26] presented a graph-based solution for the design of an SBS/RS, while Ha and Chae [27] presented a method to determine the appropriate number of shuttles to be introduced in a system, based on the travel time estimation. The queueing network model approach to estimate SBS/RS performance has been undertaken by Tappia et al. [28].

A synthesis of the existing works is provided in Table 1. Howevre, as discussed, all the researches presented above take into account single- or double-depth racks. Nonetheless, as introduced in Section 1, one strength of AVS/RS is the capability to feed multi-depth racks with an arbitrary number of UL positions for each channel. At the state of the art, this feature has been considered only by Manzini et al. [29]. Further, all the introduced papers rely on the assumption that ULs are randomly stored in the rack, although Ekren et al. [30] demonstrated that the criteria adopted for storage and retrieval significantly affect system performance.

In a recent work, D'Antonio et al. [31] proposed an analytical approach capable to evaluate the performance of an AVS/RS feeding a multi-depth rack by modelling also the criteria used for ULs storage and retrieval, defined in form of probability distributions. The model is able to assess SC, DC and Multi-Command (MC) cycles, in which an arbitrary number of ULs is involved. In that work, only the simplest AVS/RS configuration has been considered, which is made of one lift, one shuttle and one satellite. The present research aims to go a step forward by modelling more complex systems consisting of a single lift and an arbitrary number of vehicles for ULs storage and retrieval acting in a tier-to-tier configuration. The modelling approach is based on the former work; nonetheless, the presence of multiple shuttles in the system poses additional difficulties to be solved, as explained in the next sections.

3 Description of the system

As introduced in Section 1, the AVS/RS targeted by this work is a system made of three types of vehicles integrated with each other:

- the lift performs vertical movements and provides access to the different tiers of the rack;
- the shuttles perform the movements along the aisle of the operating tier;
- the satellites autonomously move through the channels of the rack to store and retrieve ULs.

Table 1 State of the art in the field of AVS/RS performance evaluation. Abbreviations: TC = Tier-captive; TT = Tier-to-tier; SC = Single Command; DC = Dual Command; CT = Cycle Time; TH = Throughput; u = machines utilization.

Authors	Configuration	Approach	S/R criterion	Rack depth	Considered cycles	Model output
Malmborg (2002) [2]	TT AVS/RS	State equation modeling	Random	Single	SC/DC	CT, TH, u
Malmborg (2003) [5]	TT AVS/RS	State equation modeling	Random	Single	SC/DC	CT
Kuo et al. (2007) [6]	TT AVS/RS	Queue network	Random	Single	SC	CT
Fukunari, Malmborg (2008) [7]	TT AVS/RS	Nested queues	Random	Single	SC/ DC	CT
Fukunari, Malmborg (2009) [8]	TT AVS/RS	Network queuing	Random	Single	SC/DC	CT, u
Zhang et al. (2009) [9]	TT AVS/RS	Queue network	Random	Single	SC	CT
Roy et al. (2012) [10]	TT AVS/RS	Queue network	Random	Single	SC	CT, u
Roy et al. (2015) [11]	TT AVS/RS	Queue network	Random	Single	SC	CT, u, TH
Roy et al. (2017) [12]	TT AVS/RS	Queue network	Random	Single	SC	CT, u, TH
Epp et al. (2017) [13]	TC AVS/RS	Queue network	Random	Single	SC	CT
Ekren et al. (2009) [14]	TC AVS/RS	Regression analysis, simulation based	Random	Single	SC	CT
Marchet et al. (2012) [16]	TC AVS/RS	Analytical model and queueing network	Random	Single	SC	CT, u
Lerher et al. (2015) [18]	TC SBS/RS	Analytical model	Random	Single	SC/DC	CT
Lerher et al. (2016) [19]	TC SBS/RS	Analytical model	Random	Double	SC/DC	CT, TH
Lerher et al. (2018) [20]	TC SBS/RS	Analytical model	Random	Single	SC/DC	CT, TH
Ekren et al. (2018) [21]	TC SBS/RS	Analytical model	Random	Single	SC/DC	CT, Energy
Borovinsek et al. (2017) [22]	TC SBS/RS	Analytical model	Random	Single	SC/DC	CT, TH, Energy
Ning et al. (2016) [23]	TC SBS/RS	Simulation	Random	Single	SC	CT, TH
Lerher et al. (2015) [24]	TC SBS/RS	Simulation	Random	Single	SC/DC	CT, TH
Lerher et al. (2017) [25]	TC SBS/RS	Simulation	Random	Single	SC/DC	CT, TH
Ekren et al. (2017) [26]	TC SBS/RS	Graph-based	Random	Single	SC/DC	CT, TH
Ha and Chae (2019) [27]	TC SBS/RS	Analytical model	Random	Single	SC/DC	CT, TH
Tappia et al. (2017) [28]	TC SBS/RS	Queue network	Random	Deep-lane	SC/DC	CT
Manzini et al. (2016) [29]	TC AVS/RS	Analytical model	Random	Deep-lane	SC/DC	CT
D'Antonio et al. (2018) [31]	TT AVS/RS	Analytical model	User-defined	Deep-lane	MC	CT, TH
Present work	TT AVS/RS	Analytical model	User-defined	Deep-lane	MC	CT, TH

The configuration considered in the present research is a tier-to-tier system made of a single lift and an arbitrary number of shuttles; each shuttle is also equipped with a satellite. Therefore, shuttles deploy the lift to change the operating level as well as satellites use shuttles to change the operating channel. The system is completed by one or several bays, which act as the interfaces between the AVS/RS and the surrounding environment: they are the place where ULs to be stored are queued and picked by the AVS/RS, and where retrieved ULs are left from the warehousing system. A schematic representation of the rack and the AVS/RS is shown in Fig. 1. In order to store an UL into the rack, the following operations must be performed:

1. the UL is placed in the bay;
2. the UL is loaded above the satellite which, in turn, joins the shuttle. Then, the two vehicles move on the lift;
3. the lift moves towards the target tier, then the shuttle leaves the lift;
4. the shuttle travels through the aisle to achieve the target channel, then the satellite leaves the shuttle;
5. the satellite moves along the channel towards the target position, chosen according to a LIFO (Last In First Out) policy;
6. the satellite unloads the UL.

Since the term 'cycle' implies that the initial and final positions of the vehicles must be the same [32], the above list is also performed backwards to carry each vehicle to the initial position. The retrieval task is performed symmetrically. One can observe that after operation (3) the lift is idle: in the AVS/RS made of several shuttles targeted by this work, the lift can serve another shuttle while the first vehicle is performing operations (4) to (6) and vice versa. Nonetheless, for safety reasons, the AVS/RS control system has to ensure that no more than a single shuttle can travel on the same tier at the same time. In a similar manner, the shuttle is idle in the time necessary for operations (5) to (6) and vice versa. Therefore, in case another UL needing to be stored on the same tier is already on the elevator, the shuttle can deploy this waiting time to move back to the lift, load the second UL, move back to the former channel and join the satellite to perform the next storage activity. The capability to simultaneously perform such operations enables to increase both the capacity and the flexibility of the warehousing system.

4 Description of the variables

The mathematical model for cycle time estimation requires a number of variables that can be grouped in the following categories:

- system parameters: rack topology, vehicles performance;
- operational parameters: cycle type, S/R criteria, movements.

Each category of parameters is discussed in detail in the following sub-sections.

4.1 Rack topology

Without loss of generalization, the rack is supposed to be symmetrical in each direction: all the tiers have the same number of channels, and all the channels have the same number of UL positions. We define:

- N_x: the number of channels that can be accessed on each tier;
- N_y: the number of tiers in the rack;
- N_z: the number of UL positions in each channel.

The capacity of each tier is therefore provided by the product $N_x \cdot N_z$, while rack capacity is given by the product $N_x \cdot N_y \cdot N_z$. For each storage position, the (x, y, z) coordinates must be known to evaluate the parameters described in the following. For sake of simplicity, we will assume that the lift travels along the positive y axis and, therefore, that bay is set in $y = 0$. Further, we also assume that the lift is placed in $x = 0$ and that aisles are placed along the positive x axis. Last, the entrance of each channel is set in $z = 0$, and the ULs storage positions are placed on the positive z axis. These assumptions are made to simplify the expression of the models, but do not restrict their validity.

4.2 Vehicles performance

The accelerations of the vehicles are denoted by (a_x, a_y, a_z). Similarly, the steady-state maximum speeds are denoted by (v_x, v_y, v_z). These properties and the rack topology parameters enable to evaluate the time necessary to reach each rack position. In particular, we denote by:

- $t(x_i)$, $\quad i = 1, \ldots, N_x$: the time necessary for the shuttle to move through the aisle from the lift to the i-th channel;
- $t(y_j)$, $\quad j = 1, \ldots, N_y$: the time necessary for the lift to move from the bay to the j-th tier;
- $t(z_k)$, $\quad k = 1, \ldots, N_z$: the time necessary for the satellite to move from the channel entry to the k-th position in the channel.

Each of these times needs to consider the acceleration and deceleration transients. For the shuttle, the time necessary to achieve the maximum speed is given by $T_x = v_x/a_x$; the distance travelled in such time is $d_x = a_x T_x^2/2$. Therefore, if the distance x_i to be traveled is lower than d_x, the maximum speed is not achieved, and the travel time is given by the acceleration and deceleration times only:

$$t(x_i) = 2\sqrt{\frac{x_i}{a_x}}. \tag{1}$$

Conversely, in case $x_i \geq d_x$, the travel time is given by:

$$t(x_i) = 2\frac{v_x}{a_x} + \frac{x - 2d_x}{v_x}. \tag{2}$$

The impact of acceleration and deceleration transients can be evaluated similarly for the lift and the satellites.

4.3 Cycle type

In order to define the cycle to be evaluated, the following parameters are necessary:

- L: the number of tiers to be visited in the cycle;
- U: the number of ULs involved at each tier;
- S: the number of switches from a storage to a retrieval task at each tier;
- P: the occurrences of simultaneous travels of the satellite and the shuttle at each tier;
- N_s: the number of shuttles involved in the cycle.

The number of ULs handled at each tier is given by the following relationship:

$$U = 1 + S + P \tag{3}$$

since the ULs following the first one have two alternatives: (1) they may require the same task performed for the previous item and, to minimize the wasted times, simultaneous travels are performed by the shuttle and the satellite, leading to increase P by one unit; (2) a transition from storage to retrieval task is to be done and, in turn, S is increased by one unit.

4.4 S/R criteria

The following discrete probability distributions are defined to describe the probability of rack positions to interact with the S/R vehicles:

- $\mathbf{a} = \{a_i\} = \{\mathcal{P}(x = x_i)\}$, $i = 1, \ldots, N_x$ is the probability to deploy the i-th channel on the tier;
- $\mathbf{b} = \{b_j\} = \{\mathcal{P}(y = y_j)\}$, $j = 1, \ldots, N_y$ is the probability to deploy the j-th tier;
- $\mathbf{c} = \{c_k\} = \{\mathcal{P}(z = z_k)\}$, $k = 1, \ldots, N_z$ is the probability to deploy the k-th position within a channel.

4.5 Movements

The probability distributions $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and the time distributions $t(\mathbf{x})$, $t(\mathbf{y})$, $t(\mathbf{z})$ enable to define the weighted means representing the expected time spent by the vehicles to move in each direction:

$$x_M = \mathrm{E}\left[t\left(x\right)\right] = \sum_{i=1}^{N_x} a_i t\left(x_i\right) \tag{4}$$

$$y_M = \mathrm{E}\left[t\left(y\right)\right] = \sum_{j=1}^{N_y} b_j t\left(y_j\right)$$

$$z_M = \mathrm{E}\left[t\left(z\right)\right] = \sum_{k=1}^{N_z} c_k t\left(z_k\right)$$

Hence, x_M is the time that the shuttle requires to move from the lift towards the expected position on the aisle; y_M is the time that the lift spends moving from the bay to the expected position on vertical direction; z_M is the time necessary for the satellite to move along the channel from its entrance to the expected point. In turn, given the acceleration and the speed of the vehicles, the spatial coordinates $(\hat{x}, \hat{y}, \hat{z})$ representing the distance travelled in the times (x_M, y_M, z_M) can be evaluated by reversing equations (1)-(2).

Some further variables need to be defined. After performing a storage operation, a shuttle may need to move towards a different channel on the same level to perform a retrieval task. In this case - which is described by a unit increase of the variable S - the shuttle movement starts at the coordinate $\hat{x}$, and the expected time spent for this displacement is denoted by δx, which results from the following weighted mean:

$$\delta x = \mathrm{E}\left[t\left(\left|\hat{x} - x\right|\right)\right] = \sum_{i=1}^{N_x} a_i t\left(\left|\hat{x} - x_i\right|\right) \tag{5}$$

Similarly, the lift may need to move between different tiers without passing through the bay, to change the served shuttle; the expected duration of this travel, which starts at the coordinate $\hat{y}$, is denoted by δy:

$$\delta y = \mathrm{E}\left[t\left(\left|\hat{y} - y\right|\right)\right] = \sum_{j=1}^{N_y} b_j t\left(\left|\hat{y} - y_j\right|\right) \tag{6}$$

Finally, a variable is necessary to describe the case in which the shuttle and the satellite move simultaneously on the respective axes. The time that the two vehicles spend being uncoupled is given by the maximum duration between the two activities, which can be described through the following relation:

$$P_{xz} = \sum_{i=1}^{N_x} \sum_{k=1}^{N_z} a_i c_k \max\left[2t\left(x_i\right), 2t\left(z_k\right)\right] \tag{7}$$

5 Analytical evaluation of AVS/RS performance

The variables presented in the previous Section enable to define analytical models for AVS/RS performance based on a probabilistic approach. The models at study enable to evaluate the expected value for both the cycle time and the throughput of a system under the following assumptions:

- the AVS/RS is made of one lift and an arbitrary number of shuttles operating in a tier-to-tier configuration;
- the number of satellites in the system is equal to the number of shuttles;
- the vehicles are provided with the capability to perform simultaneous movements;
- the lift is equipped with a table capable to host an arbitrary number of ULs, while the capacity of shuttles and satellites is equal to a single UL;
- the same set of tasks is assumed to be performed at each tier involved in the examined cycle;
- the tiers to be visited are assigned as equally as possible to the shuttles in the system;
- vehicles are dynamically allocated to the tiers, and each tier is allowed to host at most a single shuttle at any time;
- the cycle begins and finishes with one shuttle in the bay;

- the rack is symmetrical: all the aisles have the same number of channels, and all the channels have the same number of UL positions; aisles are travelled in a bi-directional way;
- all the ULs have the same size;
- the bay is placed at $y = 0$, the aisle starts at $x = 0$, the entrance of each channel is set in $z = 0$.

The analytical models are presented as follows. First, a model to describe the time necessary to perform the activities scheduled on a tier is presented. Second, the model for the lift activities is introduced. Then, these results are used to define the overall model for AVS/RS cycle time and throughput. All the variables used in the models are synthesized in Table 1.

5.1 Mathematical model for tier activities

The overall expected time for the activities occurring on a tier is given by the following contributions:

- **Shuttle times**: the time x_M must be taken into account only twice, corresponding to the first and the last travels from/to the lift; other possible shuttle travels of the shuttle to or from the elevator occur simultaneously with satellite activities (see below). Furthermore, movements to change the satellite operating channel - which duration is δx - occur after each shuttle-satellite simultaneous activity (i.e. P times), and after each switch from storage to retrieval (i.e. S times).
- **Satellite times**: the time z_M must be taken into account twice (to go back and forth along the channel) only in case no shuttle-satellite simultaneous activities occur, i.e. $(U - P)$ times.
- **Simultaneous activities**: they occur P times and have duration P_{xz}.

The resulting expected time is:

$$T_{tier} = 2x_M + (P + S)\,\delta x + 2\,(U - P)\,z_M + P \cdot P_{xz}. \quad (8)$$

5.2 Mathematical model for lift activities

While a shuttle is involved in storage or retrieval activities on a tier, the lift has to serve the other $N_s - 1$ shuttles requiring to unload in the bay the ULs retrieved from the operating tier and loading further ULs to be stored in the rack. The time necessary for this operation is given by:

$$T_{lift} = (N_s - 1)\,(\delta y + 2y_M + 2Px_M) + \delta y, \quad (9)$$

Table 2 Summary of variables used in the model.

Notation	Meaning
$\mathbf{a}$	Probability to exploit each of the channels available on the x axis
a_x	Acceleration of the shuttle
a_y	Acceleration of the lift
a_z	Acceleration of the satellite
$\mathbf{b}$	Probability to exploit each of the tiers available on the y axis
$\mathbf{c}$	Probability to exploit each of the UL positions available in a channel
CT	Expected system cycle time
δx	Expected time necessary to travel between two different channels
δy	Expected time necessary to travel between two different tiers
L	Number of levels involved in the cycle to be examined
N_s	Number of shuttles involved in the cycle
N_x	Number of channels for each aisle of the rack
N_y	Number of tiers of the rack
N_z	Number of UL positions within a channel
P	Number of simultaneous operations taking place at each tier involved in the cycle
P_{xz}	Expected duration of each simultaneous operation
S	Number of switches from a storage to a retrieval task at each tier involved in the cycle
T_{lift}	Time necessary for the lift to serve all the shuttles in the system
$T_{shuttle}$	Time necessary for the shuttle to perform all the activities scheduled on a tier without needing the lift being idle
T_{tier}	Time necessary for the shuttle to perform all the activities scheduled on a tier
TH	Expected system throughput
U	Number of ULs to be stored or retrieved at each tier involved in the cycle
x_M	Expected time necessary to travel from the lift to the target channel
v_x	Maximum speed of the shuttle
v_y	Maximum speed of the lift
v_z	Maximum speed of the satellite
y_M	Expected time necessary to travel from the bay to the target lift
z_M	Expected time necessary to travel from the channel entrance to the target position

Table 3 Summary of abbreviations.

Acronym	Meaning
AS/RS	Automated Storage and Retrieval System
AVS/RS	Autonomous Vehicle Storage and Retrieval System
CC	Closest Channel
CF	Closest Floor
DC	Dual Command
LIFO	Last In First Out
MC	Multi Command
SBS/RS	Shuttle Based Storage and Retrieval System
SC	Single Command
UL	Unit Load

as for each of the $N_s - 1$ shuttles, the lift needs to travel to the proper tier (duration δy), move from the tier to the bay and vice versa ($2y_M$), and remain idle for the time necessary to support the simultaneous operations involving the shuttle and the satellite ($2x_M$, repeated P times). Last, after having served each shuttle, a displacement with duration δy is necessary to reach the first vehicle.

5.3 Mathematical model for cycle time evaluation

The first step to evaluate the expected AVS/RS cycle time is the identification of the system bottleneck in the cycle under examination. To this purpose, the following quantity can be defined:

$$T_{shuttle} = T_{tier} - 2Px_M. \tag{10}$$

It represents the time that each shuttle spends traveling on a tier performing S/R tasks, without needing for the lift being idle on the same tier. T_{tier} is reduced in presence of simultaneous operations for the shuttle and the satellite (i.e. for $P > 0$): while the satellite is storing an item in the channel, the lift is idle at the tier, waiting for the shuttle that has to pick the further UL to be stored at the same level. The same process occurs symmetrically in case of simultaneous operations during retrieval tasks.

In case $T_{shuttle} \geq T_{lift}$, the lift unloads a shuttle on a given tier and has enough time to serve all the other shuttles in the system and to move back towards the first vehicle. Eventually, it remains idle for an amount of time equal to $T_{shuttle} - T_{lift}$. In this case, the cycle time of the system is determined by the shuttle activities, which is the bottleneck of the system.

Conversely, if $T_{shuttle} < T_{lift}$, the lift is the bottleneck of the system and determines the overall cycle time. Therefore, the following, mutually alternative cases may occur:

Case 1. Cycle involving both storage and retrieval tasks, $T_{shuttle} \geq T_{lift}$. To determine the overall cycle time, two variables must be evaluated.

First, the shuttle that last terminates the cycles is identified: it is the shuttle in charge to serve the L-th tier. Given the following quantity:

$$K = \mathrm{mod}\left(\frac{L-1}{N_s}\right), \tag{11}$$

where mod$(\cdot)$ is the remainder of the ratio, the $(K+1)$-th shuttle that begins its operations is the one that terminates the cycle under consideration.

Second, the number of tiers visited by the $(K+1)$-th shuttle is evaluated:

$$N_L = \left\lceil \frac{L}{N_s} \right\rceil, \tag{12}$$

where $\lceil\cdot\rceil$ is the ceiling function.

The cycle time is given by two main contributions:

1. the time necessary to run the activities of the first K shuttles (i.e. to move from the tier to the bay and vice versa), and enable the lift to change the operating tier;
2. the time necessary for the $(K+1)$-th shuttle to visit N_L tiers, and to go back and forth from the bay after performing the assigned tasks on each tier.

$$\mathrm{CT} = K\left(\delta y + 2y_M\right) + N_L\left(2y_M + T_{tier}\right). \tag{13}$$

Case 2. Cycle involving only storage or retrieval tasks, $T_{shuttle} \geq T_{lift}$. The mathematical model is similar to the previous case; the cycle time is reduced by a quantity y_M, as in case of storage tasks, there is no need to carry the K-th shuttle to the bay to perform the last UL download: the shuttle $K-1$ remains in the bay after its last cycle. Symmetrically, in case of retrieval tasks, the first shuttle is supposed to be already on the operating tier. Hence, the cycle time is given by:

$$\mathrm{CT} = K\left(\delta y + 2y_M\right) + N_L\left(2y_M + T_{tier}\right) - y_M. \tag{14}$$

Case 3. Cycle involving both storage and retrieval tasks, $T_{shuttle} < T_{lift}$. The cycle time of the system is given by the sum of two contributions:

1. at each visited tier, the lift loads a shuttle that needs to go to the bay, unload the retrieved ULs, load the ULs to be stored, and go back to the operating tier. Here, the lift still has to wait for eventual shuttle-satellite simultaneous operations and, finally, is enabled to change the tier to be ready to serve the next shuttle;
2. for $N_s - 1$ shuttles, the time necessary for the first travel from the tier to the bay, and for the last travel from the bay to the tier must be considered. Here, N_s is reduced by one unit as at the beginning and at the end of the cycle one shuttle is supposed to be in the bay.

$$\mathrm{CT} = L\left(2y_M + 2Px_M + \delta y\right) + \left(N_s - 1\right)\left(\delta y + 2y_M\right). \tag{15}$$

Case 4. Cycle involving only storage or retrieval tasks, $T_{shuttle} < T_{lift}$. In this case, the second term in model (15) is erased, as there is no need to carry shuttles to the bay after a storage operation or, symmetrically, to carry shuttles to the tier before a retrieval task. Conversely, a term must be added to ensure that the activities occurring on the tiers are fully considered by the model: T_{tier} may be higher than the time spent by the lift to change the operating tier, load a shuttle and go to the bay.

$$\mathrm{CT} = L\left(2y_M + 2Px_M + \delta y\right) + \max\left\{T_{tier} - \delta y - y_M; 0\right\}. \tag{16}$$

A synthesis of the models for cycle time evaluation is provided in Table 4.

5.4 Mathematical model for throughput evaluation

The expected throughput of the system is given by the following ratio:

$$TH = \frac{(U \cdot L)}{CT} \tag{17}$$

The overall operational logic is represented in Table 5.

6 Case study and models validation

The mathematical models presented in the previous Section have been validated through a simulation plan that considers a wide variety of configurations. The simulations enabled to collect the duration of each cycle; the average value was then compared with the estimation provided by the mathematical model. A high-level description of the simulation model is provided in the appendix. To assess the performance of the model in the widest possible number of scenarios, the following five factors are varied in the investigations:

1. the layout of the rack;
2. the type of cycle to be performed;
3. the criteria used to store and retrieve the ULs;
4. the number of shuttles in the AVS/RS;
5. the number of tiers visited in the analyzed cycle.

Each experiment consisted in storing and retrieving a sequence of 20,000 ULs. This number has been chosen as it is around an order of magnitude higher than the capacity of the considered racks and, in turn, enables to get statistically meaningful results.

In order to be closer to realistic deployment scenarios, the rack is assumed to manage 4 different types of items. Therefore, each UL in the sequence is provided with a type randomly assigned according to a discrete uniform distribution.

The levels assigned to each of these factors are detailed in the following sub-sections. To test each combination of the levels, a full experimental plan made of 1,980 scenarios has been designed. To evaluate the repeatability of the system and ensure the robustness of the results, each scenario is simulated ten times: for each repetition, a different initial rack content and a different S/R sequence were adopted. To this purpose, ten initial rack contents have been generated a-priori with a fill ratio close to 50%. Each of them is associated to a different S/R sequence with a ratio between storage and retrieval orders kept close to 1 in order to simulate a steady state scenario. The ten initial conditions and S/R sequences are used in all the scenarios. Therefore, an overall amount of 19,800 experiments has been performed.

Simulations have been run through a Matlab routine properly developed. During the run, each channel is enabled to store a single type of UL in a dynamic manner: if the channel is empty, any class of item can be stored; otherwise, the units stored in the channel must be of the same type.

Two kinds of output are then extracted from the simulations. First, data concerning the S/R positions are collected to define the discrete probability distributions **a**, **b**, **c** and, in turn, enable the analytical cycle time evaluation. Second, the time necessary to perform each cycle is collected in the simulations; these values are then averaged over each simulation scenario to perform a comparison with the results provided by the mathematical models.

6.1 Rack layouts

Three warehouse layouts, already studied in [31] and inspired to real projects dealt by an industrial partner, have been used for this purpose: they exhibit different scale-factor values (i.e. the ratio between the length and the width of the rack). In each layout, rack height is equal to 10 tiers, and the capacity is close to 2,000 ULs. A synthesis of the geometrical rack properties and the features of the vehicles is provided in Table 6.

6.2 Type of cycle

Four sets of activities to be made on each tier are tested to simulate different operating conditions and evaluate the capabilities of the analytical models. SC and DC cycles are tested first; then, the investigation is extended to account for cycles involving up to 4 ULs per each tier, in order to evaluate scenarios capable to fully exploit the technical capabilities of the AVS/RS under

Table 4 Summary of the analytical models developed for cycle time estimation. S=Storage, R=Retrieval.

Use case	Condition	Tasks	Model
1	$T_{shuttle} \geq T_{lift}$	S+R	$\text{CT} = K(\delta y + 2y_M) + N_L(2y_M + T_{tier})$
2	$T_{shuttle} \geq T_{lift}$	S or R	$\text{CT} = K(\delta y + 2y_M) + N_L(2y_M + T_{tier}) - y_M$
3	$T_{shuttle} < T_{lift}$	S+R	$\text{CT} = L(2y_M + 2Px_M + \delta y) + (N_s - 1)(\delta y + 2y_M)$
4	$T_{shuttle} < T_{lift}$	S or R	$\text{CT} = L(2y_M + 2Px_M + \delta y) + \max\{T_{tier} - \delta y - y_M; 0\}$

Table 5 Summary of the operational logic of the developed model.

Task	Variables	References
1. Configure the rack	x, y, z, N_x, N_y, N_z	—
2. Configure vehicles and fleet size	$a_x, a_y, a_z, v_x, v_y, v_z$	Eqs. (1), (2)
3. Configure cycle type	L, S, P, N_s	Eq. (3)
4. Configure storage/retrieval criteria	$\mathbf{a}, \mathbf{b}, \mathbf{c}$	—
5. Evaluate expected duration of elementary movements	$x_M, y_M, z_M, \delta x, \delta y, P_{xz}$	Eqs. (4), (5), (6), (7)
6. Evaluate expected duration of tier and lift activities	T_{tier}, T_{lift}	Eqs. (8), (9)
7. Decide among use cases 1-4 and apply the appropriate cycle model	CT	Table 4
8. Evaluate system throughput	TH	Eq. (17)

Table 6 Summary of the parameters for the rack layouts and the vehicles performances considered in this work.

		Layout 1	Layout 2	Layout 3
UL size [m]	x		1.5	
	y		2.0	
	z		1.2	
Rack size [nr. of ULs]	x	11	22	40
	y	10	10	10
	z	19	9	4
	Total	2090	1980	2000
Rack size [m]	x	16.5	33.0	60.0
	y	20.0	20.0	20.0
	z	22.8	10.8	4.8
Scale factor [-]		0.73	3.05	12.5
Vehicles acceleration [m/s^2]	x		0.5	
	y		0.3	
	z		0.5	
Vehicles max. speed [m/s]	x		2.0	
	y		0.2	
	z		1.2	

Table 7 Summary of the parameters describing the tasks considered in this work performed on rack tiers. Rack sizes are inspired to projects dealt by an industrial partner. Vehicles performances are taken from systems available on the market.

ID cycle	Tasks	Parameters U	S	P
1	1 storage	1	0	0
2	1 storage, then 1 retrieval	2	1	0
3	2 storages, then 1 retrieval	3	1	1
4	2 storages, then 2 retrievals	4	1	2

investigation. A detailed summary of the parameters describing each scenario is provided in Table 7.

6.3 Storage and retrieval criteria

Three different criteria are chosen to test the analytical model in different deployment scenarios:

1. Closest Channel (CC). The position is selected according to the following hierarchy: (a) the channel closest to the lift hosting the same type of item; (b) the tier closest to the bay.
2. Closest Floor (CF). The position of the UL is selected according to the following hierarchy: (a) the tier closest to the bay; (b) the channel closest to the lift hosting the same type of item;
3. Random. The position of the UL is chosen randomly, according to a uniform distribution.

The random criterion is commonly used in literature, whilst the former two ones are in fact implemented by AVS/RS providers, and are chosen according to the capabilities (acceleration, maximum speed) of the vehicles: CF minimizes the deployment of the lift, while CC minimizes the shuttle travels.

6.4 Number of shuttles and visited tiers

The tested number of shuttles N_s in the system varies from 1 to 5, provided that each tier can host at most one shuttle for safety reasons. The maximum value $N_s = 5$ has been chosen as a tier-to-tier configuration is dealt: in case of a larger fleet, some shuttles would never change the operating tier. Similarly, the number of levels L involved in the tested cycles varies from 1 to 5, provided the constraint $L \geq N_s$.

Table 8 Analysis of variance for the relative difference of cycle time between the analytical model and the simulations.

Factor	Degrees of Freedom	F	p
Layout	2	285.30	< 0.001
Tier activities	3	1143.82	< 0.001
Criterion	2	849.20	< 0.001
Nr of shuttles	4	2987.57	< 0.001
Tiers	4	366.05	< 0.001
Error	5384		
Total	5399		

7 Results

General considerations. For each experiment, the relative difference between the cycle time estimation provided by the models in Table 2 and the average value resulting from the corresponding simulation has been evaluated. The distribution of the results is shown in Fig. 2a. Here, positive values mean that the analytical model is overestimating the cycle time. The histogram is not symmetric: the number of positive values is higher than the negative values, meaning that - in general - the model tends to overestimate (i.e. to provide a conservative estimation) the cycle time. Nonetheless, model deviation is within ±5% in the 53.5% of the cases. The 84.3% of the scenarios exhibits a difference smaller than ±10%, while only in 1 case a difference higher than 20% has been found. The number of repetitions for each configuration was enough to guarantee statistically meaningful conclusions: the ratio between CT estimation difference and the expected CT was at most equal to 10%; in more than 90% of the cases, such difference was smaller than 7%. In the following, the impact of each varied factor is examined. To support this step, an analysis of variance (ANOVA) has been performed: all the five factors varied in the experiment were found to be significant for the deviation of the analytical model from the simulations, with $p < 0.001$ and $R^2 \approx 77\%$. The results are shown in Table 5.

Impact of rack layout. The boxplot in Fig. 2b shows the distribution of the model deviation in the different layouts. In all the three layouts, the model tends to overestimate the cycle time. The impact of this trend is higher as the scale factor increases: layout 1 exhibits the highest median value and owns the highest scale factor. Further, layout 3 (characterized by the smallest scale factor) also exhibits the smallest variability.

Impact of type of cycle. The boxplot in Fig. 2c shows that the model tends to overestimate (around 3-5%) the cycle time for single and dual command cycles. Conversely, the median relative difference for cycle time is lower than 1% for cycles 3-4. For all the cycles, the range represented by the whiskers is in the order of 24-26%.

Impact of the S/R criterion. The boxplot in Fig. 2d shows that when the criterion Closest Channel is adopted, the mathematical model tends to most overestimate the cycle time; nonetheless the denser quartiles are Q1-Q2. Conversely, in case the criterion Closest Floor is used, the denser quartiles are Q3-Q4. The Random criterion leads to an almost symmetric distribution (with respect to the median value) of the cycle time deviation provided by the model.

Impact of the number of shuttles. The boxplot in Fig. 2e shows that in case a single shuttle is used the model exhibits a deviations range close to 20%. This value is maximum when 2 shuttles are available in the system, mainly because of two reasons: (i) the model estimates expected waiting times for the vehicles, but different behaviours may occur within each S/R cycle; (ii) in this case, a high variability of scenarios occurs: the two shuttles may have to serve 2 to 5 tiers. The number of scenarios and, hence, the variability, is reduced as the number of shuttles increases, due to the constraint $L \geq N_s$, while in case $N_s = 1$ the lift is always available for the single shuttle and no waiting times for synchronization arise.

Impact of the number of visited tiers. The boxplot in Fig. 2f shows that the model has a high reliability for cycles involving a single tier: the average deviation provided by the model is always within ±3. Conversely, as different shuttles operating at different tiers have to share a single lift, model reliability decreases: the error affecting the provided estimations may rise up to 20%.

The plots in Fig. 3 synthesise the average throughput values obtained in the tested scenarios. Fig. 3b shows the importance of modelling the latest technological advances in AVS/RS. The exploitation of simultaneous activities for shuttles and satellites (cycles 3-4) leads to a significant increase in the throughput values corresponding, in turn, to a high AVS/RS capability to deal with variable scenarios. Also the storage and retrieval criteria play a key role (see Fig. 3c): the random storage exhibits the poorest performance. By switching to the Closest Floor criterion, the throughput performance can be almost doubled. Nonetheless, there is not a linear correlation between the number of shuttles in the system and the overall performance. Increasing the number of shuttles does not necessarily correspond to throughput improvement. Fig. 3d shows that the best performance is achieved when the AVS/RS is made of 2-3 shuttles, which is the configuration resulting in the best compromise for lift and shuttles saturation. Conversely, as the number of shuttles increases, the lift acts as a bottleneck and the shuttles spend time being idle and waiting for the lift.

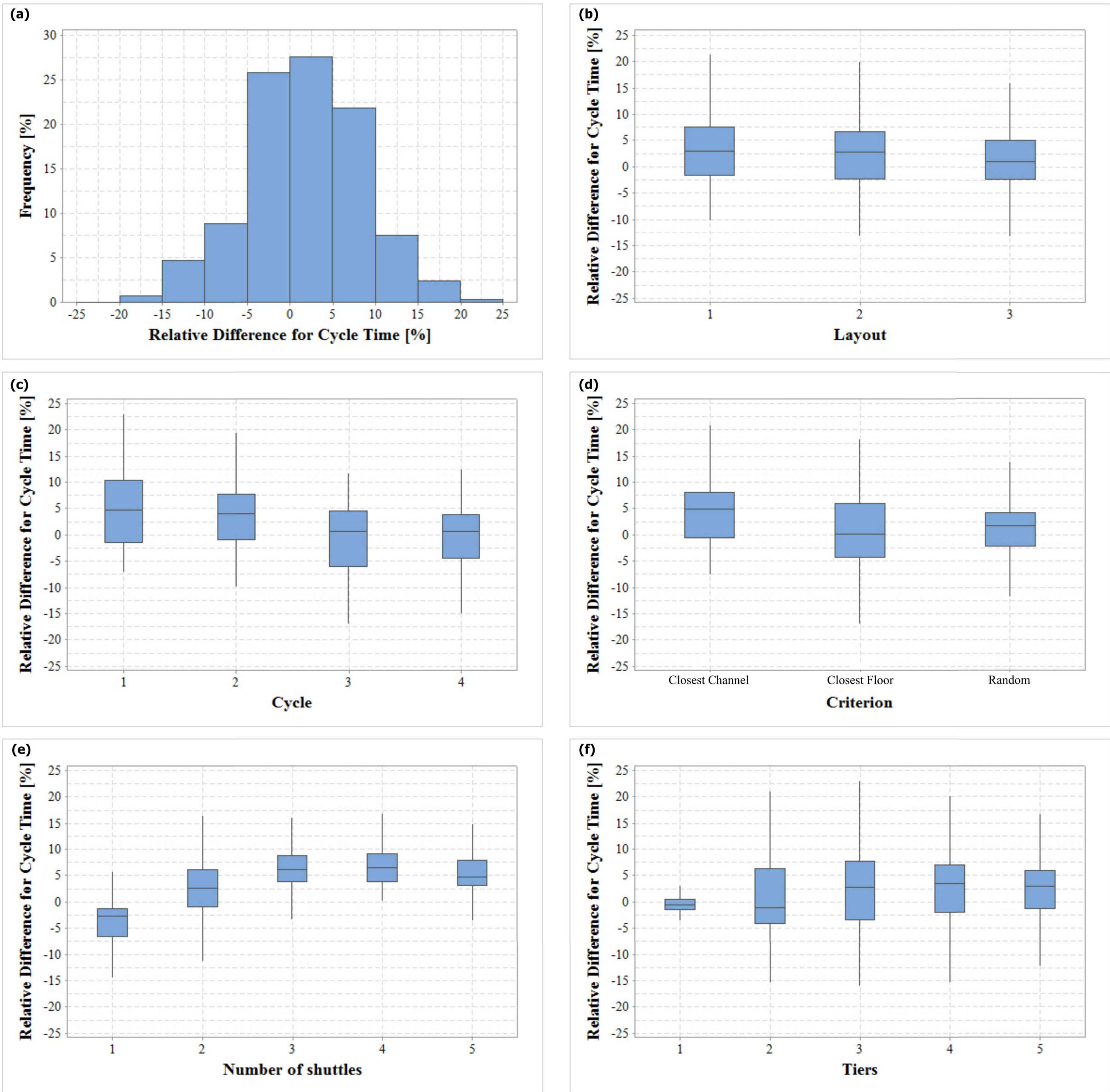

Fig. 2 Graphical representation of the relative differences for cycle time between the analytical model and the simulations.

8 Conclusions and perspective work

In the present research paper, novel analytical models for the evaluation of AVS/RS cycle time and throughput have been presented. The models are able to assess the performance of a tier-to-tier AVS/RS made of a lift and an arbitrary number of shuttles operating on a rack with arbitrary lane depth, a kind of system still under-investigated in literature. The modelling approach was inspired to [31], but a step forward has been made to model a multi-shuttle system: since the lift has to serve multiple vehicles, different scenarios have to be considered, compared to the simpler system already investigated.

Further, the models enable to assess the impact of an arbitrary S/R criteria, while most of the state-of-the-art literature still assumes that S/R tasks are assigned according to random, uniform distributions.

The model has been validated against simulations where the rack fill rate was kept close to 50%. Of course, in realistic conditions this value might be different as well as it may vary over time. However, this choice does not restrict the validity of the model, as it only impacts the probability distributions a, b, c: for example, in the case of Closest Channel criterion, as the fill rate increases, it may become 'more difficult' to find a place for the UL in a channel close to the lift.

8.1 Model deployment

The approach presented in this work can support the activities of both designers and end-users of AVS/RS. First, in the design phase, the performance of the system can be assessed in a variety of scenarios: provided a given rack structure, changes in vehicles speed and accelerations, variations in the vehicles fleet, adaptation of storage and retrieval criteria can be easily evaluated for an arbitrary cycle. Thus, customers can be provided with a tailored estimation of the AVS/RS capabilities. Second, users of such system can evaluate the effect of an operative strategy (storage and retrieval criteria, type of cycle) before its implementation: a preliminary study is provided in [33].

For the sake of models validation, here the probability distributions **a**, **b**, and **c** have been extracted from simulations. The plots in Fig. 4 show the distributions used in the present work with the criteria Closest Channel, Closest Floor and Random. Nonetheless, either in the AVS/RS design or deployment phases, **a**, **b**, **c** may be any set of weights with a (normalized) sum equal to 1. During the design phase, discrete probability distributions can be used based on the scenario to be assessed: for example, uniform distributions to model random storage/retrieval or weights with a shape similar to geometrical distribution can be used to represent the Closest Channel or the Closest Floor approaches. In case of an existing system, an alternative way can consist in extracting data on storage/retrieval tasks for a statistically significant time span.

The results discussed in Section 7 show that the models presented in this paper provide a deviation from the simulated behaviour that is in most of the cases within ±10% and rarely achieves ±20%. Further, the models tend to overestimate cycle time, thus provide a conservative estimation. Given these results, we can state that the analytical models represent a useful tool for the evaluation of system performance. They can be easily implemented into an electronic spreadsheet and promote quick performance evaluation over a wide range of scenarios in the early-design phase. The results presented in Fig. 3 highlight the importance of the original contribution presented in this work: the criteria adopted for storing and retrieving ULs, as well as the flexibility that can be achieved by acting on the operating principles, such as simultaneous operations, are proven to significantly affect AVS/RS performance. Therefore, an appropriate modelling is necessary.

8.2 Discussion on model assumptions

In the beginning of Section 5, the list of model assumptions has been provided. Here, a short discussion concerning their relaxation is provided.

Composition of the system. The modeled AVS/RS consists of a single lift and an arbitrary number of shuttles. In case two or more lifts are adopted in the system, two possibilities arise:

- in case each lift serves an independent set of aisles, the overall system can be modeled as a set of independent AVS/RSs;
- in case each aisle is served by more than one lift, the model needs some adaptations: for each rack position (x_i, y_j, z_k), the storage/retrieval probability changes according to the involved lift. Therefore, the probability distributions **a**, **b**, **c** will consist in bidimensional arrays, where the second dimension is equal to the number of lifts in the AVS/RS.

Further, the number of satellites was assumed to be equal to the number of shuttles. From the technological point of view, the number of shuttles and satellites in the system are totally independent. However, operational criticalities may arise in case this ratio is not 1:1, as waiting times for satellites (which can autonomously move only within a rack channel) may significantly arise as well as not-in-service shuttle travels necessary to switch the served satellite. In other words, a customer would be required to pay money for vehicles with low efficiency. Conversely, the capability of shuttles and satellites (available on the market) to simultaneously move on different directions enables to improve the overall efficiency without buying a huge fleet of vehicles. From the model perspective, an adaptation is necessary to consider the aforementioned additional shuttle travels.

Capabilities of the system. Shuttles and satellites are assumed to be provided with the capability to perform simultaneous movements; if this is not the case, the model still holds as $P = 0$ disables such capability. Further, the capacity of shuttles and satellites is assumed to be equal to a single UL. This is mandatory for the satellite: as it needs to enter within the channel rack, it cannot host more than a single UL. Conversely, shuttles capable to host more than a single UL can be designed: in this case, the equation for T_{tier} needs to be adapted.

Tasks to be performed. The same set of tasks is assumed to be performed at each tier involved in the examined cycle: since the purpose of this work is to look for average values of cycle time, this assumption is reasonable. Considering specific tasks different for each tier would go to the direction of task scheduling, which is out of the scopes of this work. Another assumption is made concerning tiers choice: the tiers to be visited are assigned as equally as possible to the shuttles in the system. Over the long term, this assumption is reasonable, as in a tier-to-tier configuration there is no meaning to sistematically prefer a shuttle to another one. Further, each tier is assumed to host at most a single shuttle at any time: this action is usually adopted by AVS/RS manufacturers for safety reasons, in order to avoid collisions. However, model adaptation is required if this is not the case.

Layout. The rack is assumed to be symmetrical: all the aisles have the same number of channels, and all the channels have the same number of UL positions; aisles are travelled in a bi-directional way. Distances between tiers and channels are considered in the travel times along each direction $(x_M, \delta x, y_M, \delta y)$; asymmetries would just be absorbed in the evaluation of such expected values, without any further impact on the model.

All the ULs are assumed to have the same size. In case of ULs with different size, some positions could be not accessible for some UL types: this will be taken into account in the probability distributions **a**, **b**, **c**, without any further impact on the model.

8.3 Future improvements

Although the obtained results are promising, there is further room for models improvement. First, the attention paid to energetic aspects is increasing also in warehousing. Therefore, in future work the model will be enriched with the estimation of the expected energy needed to perform a given cycle and for each storage or retrieval task. Further, besides the expected values of such performances, the evaluation of a variability measure (e.g. the standard deviation) may be desirable to assess the reliability of the evaluated information. Finally, the model presented here is capable to consider only symmetric cycles, i.e. cycles in which a given set of activities is performed in all the involved tiers; the capability to evaluate asymmetric cycles would improve the flexibility of the tool.

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A Description of the simulation model

This section contains a high level description of the simulation model used for validation of the developed approach.

A.1 Initialization model

A first model has been developed to generate 10 datasets to be used in the different repetitions of each configuration to be simulated: such datasets contain the rack content to be loaded at the beginning of the simulation and a list of ULs to be stored and retrieved.

```
define rackCapacity = N_x*N_y*N_z
for i=1:numberOfRepetitions
   generate the empty rack structure
   set storageCriterion: Closest Channel
   set retrievalCriterion: Random
   for j=1:rackCapacity
      generate the j-th UL
      provide the j-th UL with a random UL type
      define the j-th UL position in the rack
         accordingly to the storageCriterion
      update rack content
   end
   for j=1:fillRate*rackCapacity
      require retrieval of a UL with a random type
      select the UL to be retrieved accordingly to
         the retrievalCriterion
      make the rack position free and update rack
         content
   end
   for j=1:numberOfInvolvedULs
      generate a new UL to be stored
      generate a new UL to be retrieved
      provide the two ULs with a random UL type
   end
   save i-th rack content
   save i-th list of storage and retrieval ULs
end
```

A.2 Simulation model

Here, the model that simulates ULs storage and retrieval is presented: the results of the former model are used here as an input; the output mainly consists in the evaluation of the average cycle time and an array (which size is equal to rack capacity) listing the number of storage/retrieval activities occurred in each rack position: this is the basis to evaluate the probability distributions **a**, **b**, **c**.

```
load rack configuration and initial content
load the list of storage and retrieval ULs
load simulation configuration: storageCriterion,
   retrievalCriterion, N_S, L, S, P, U
define numberOfCycles = 20000/U
define arrayOfInteractions = zeros(size: rackCapacity)
for i=1:numberOfCycles
   identify the U items in the list involved in
      the actual cycle
   decide the rack positions of ULs to be stored
      accordingly to the storageCriterion
   decide the rack positions of ULs to be retrieved
      accordingly to the retrievalCriterion
```

```
   start timeCounter
   run vehicles movements to store/retrieve the U
      items, accordingly to cycle specifications and
      to the defined rack positions
   end timeCounter
   set cycleDuration(i) = timeCounter
   increase by one unit the appropriate value of
      arrayOfInteractions
end
evaluate a, b, c based on arrayOfInteractions
evaluate avgCicleDurationSimulation = mean(...
   cycleDuration(1:numberofCycles)
```

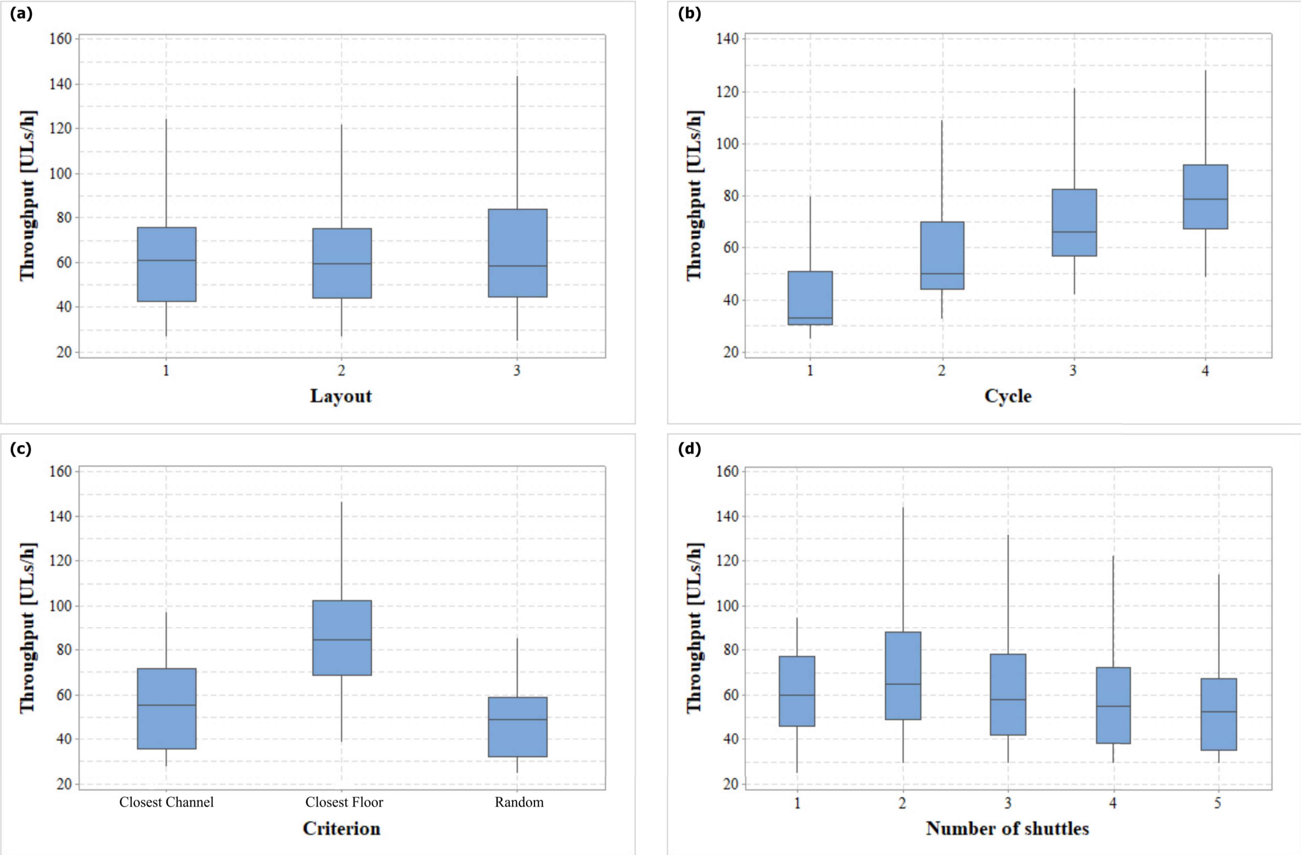

Fig. 3 Graphical representation of the throughput achieved by the AVS/RS.

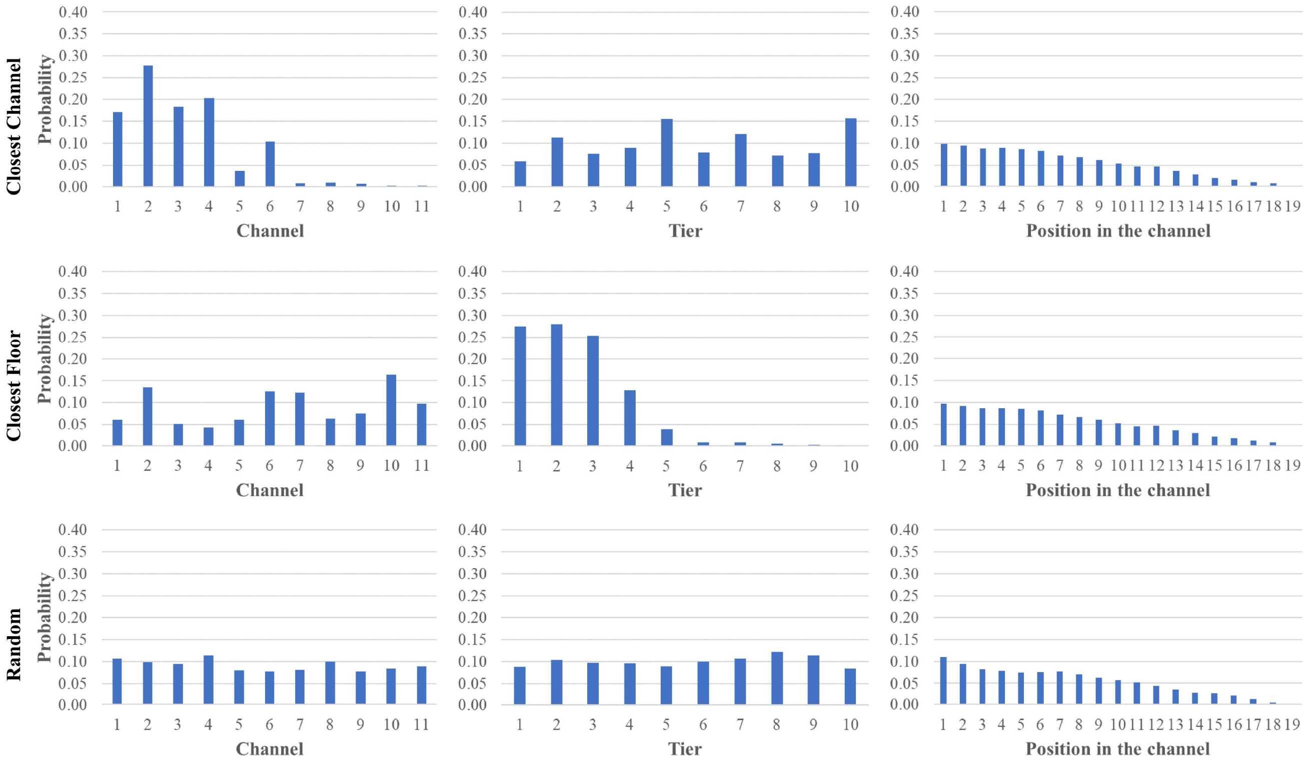

Fig. 4 Examples of probability distributions describing the storage/retrieval criteria for Layout 1 used in this work.